A New Microscopic Approach to the Rotational Intensity Relations
— Application of the High-Spin Cranking Formalism —

Yoshifumi R. Shimizu and Takashi Nakatsukasa†

Department of Physics, Kyushu University, Fukuoka 812, Japan

† AECL, Chalk River Laboratories, Chalk River, Ontario K0J 1J0, Canada

Abstract

The intensity relations for electromagnetic transition rates in the rotational coupling scheme have been a basic tool to understand the properties of nuclear collective rotations. In particular the correction terms to the leading-order relation give the response of the intrinsic motion to the Coriolis and centrifugal forces. We propose a simple and systematic method to calculate the matrix elements of intrinsic operators entering the generalized intensity relations that uses the microscopic cranking formalism. Examples are given to show the usefulness of the method.

§1 Introduction

The cranking model is a powerful tool to study rapidly rotating nuclei: nonlinear effects such as the alignment of quasiparticles can be selfconsistently described from a microscopic view point. An apparent drawback, however, is the semiclassical treatment of the nuclear rotational motion which assumes rotation around one of the principal axis (usually that perpendicular to the symmetry axis). This drawback is not necessarily a serious problem at very high spin, but it can be a severe limitation at low spin or in cases where the K-quantum number is comparable to the total spin.

While the assumption of rotation around a principal axis can be lifted within the semiclassical approximation by considering tilted-axis rotation,[1] the precise angular-momentum algebra (i.e. the effects of Clebsch-Gordan coefficients) has been only taken into account by e.g. angular-momentum projection. On the other hand, in the unified model,[2] the rotor part of the wave functions exactly takes care of the angular-momentum algebra. Despite recent efforts (e.g. Ref. 3)) a microscopic foundation for the model remains elusive, nevertheless, it has been a basic and useful tool to analyze not only energy

spectra but also electromagnetic transition rates. The problem of the model is that there has been no systematic way to calculate the intrinsic matrix elements from the microscopic view point.

In this paper we propose a systematic and yet simple method to calculate the intrinsic matrix elements entering into the generalized intensity relations (GIR).[2] This is done by starting from the semi-microscopic cranking formalism of Ref. 4) and by taking a heuristic "quantization" procedure. It should be noted that the method of GIR is essentially perturbative with respect to angular momentum[2] : it is therefore applicable at low spin, and in this respect, the method is complementary to the usual cranking model which is useful at high spin.

The paper is organized as follows: In §2 the basic formalism is presented. Explicit formulae for the $M1$ and $E\lambda$ intraband and interband transitions, obtained as a result of application of the basic formalism, are presented in §3. Some examples of numerical calculations for the quadrupole- and octupole-vibrational bands are discussed in §4. Section §5 is devoted to the concluding remarks.

§2 Basic Formalism

In this paper we assume that nuclei are axially symmetric in their ground states and that signature symmetry is satisfied.*)

The GIR is obtained by taking the matrix element of the transition operator in the laboratory frame with respect to the unified model wave functions, Eq.(4-19) in Ref.2),

$$\Psi_{KIM}(\Theta, q) = \sqrt{\frac{2I+1}{(1+\delta_{K0})16\pi^2}} \Big(\mathcal{D}^I_{MK}(\Theta)\Phi_K(q) + \mathcal{R}-\text{conj.} \Big), \quad (K \geq 0). \tag{2.1}$$

In this description, the coupling between rotational and intrinsic motion is not seen explicitly, but it appears in the angular-momentum dependence of the intrinsic operators. The leading-order relation and the higher-order corrections are derived by expanding the intrinsic operator with respect to the (intrinsic) angular-momentum operators. For example,

*) This does not exclude the possibility of the rotationally induced triaxial deformation at higher spin. We only assume that the deformation is axially symmetric at rotational frequency $\omega_{\rm rot} = 0$ in the cranking terminology.

the operator for the K-allowed transitions ($\Delta K \leq \lambda$) is expressed as

$$\begin{aligned}\mathcal{M}(\lambda\mu) &= \sum_{\nu} \mathcal{M}(\lambda\nu; I_{\pm}) \mathcal{D}^{\lambda}_{\mu\nu}(\Theta) \\ &= \sum_{\nu} \widehat{m}^{(0)}_{\lambda\nu}\, \mathcal{D}^{\lambda}_{\mu\nu} + \sum_{\nu} \widehat{m}^{(+1)}_{\lambda\nu}\, \frac{1}{2}\{I_+, \mathcal{D}^{\lambda}_{\mu\nu}\} + \sum_{\nu} \widehat{m}^{(-1)}_{\lambda\nu}\, \frac{1}{2}\{I_-, \mathcal{D}^{\lambda}_{\mu\nu}\} + \cdots, \quad (2.2)\end{aligned}$$

where the $\widehat{m}$'s are intrinsic operators which do not depend on the rotational variables $(\Theta, I_{\pm})$.* The transitions with definite $\Delta K \equiv K_{\rm f} - K_{\rm i}$ are obtained within the lowest-order corrections by the operator,

$$\begin{aligned}\mathcal{M}(\lambda\mu)_{\Delta K} =& \widehat{m}^{(0)}_{\lambda\Delta K}\, \mathcal{D}^{\lambda}_{\mu\Delta K} + \widehat{m}^{(+1)}_{\lambda\Delta K+1}\, \frac{1}{2}\{I_+, \mathcal{D}^{\lambda}_{\mu,\Delta K+1}\} + \widehat{m}^{(-1)}_{\lambda\Delta K-1}\, \frac{1}{2}\{I_-, \mathcal{D}^{\lambda}_{\mu,\Delta K-1}\} \\ &+ (1-\delta_{\Delta K,0})\, \mathcal{R}-\text{conj.}, \quad (2.3a)\end{aligned}$$

where for simplicity we omit the so-called signature-dependent (decoupling) terms[2] in the following. Such terms can be easily evaluated if necessary. Then the reduced transition amplitude is written as

$$\begin{aligned}\mathcal{T}(\lambda : \mathrm{i} \to \mathrm{f}) &\equiv \langle K_{\rm f} I_{\rm f} || \mathcal{M}(\lambda) || K_{\rm i} I_{\rm i} \rangle / \sqrt{2I_{\rm i}+1} \\ &= C_{\rm if} \left[(\mathrm{f}|\widehat{m}^{(0)}_{\lambda\Delta K}|\mathrm{i})\, \langle I_{\rm i} K_{\rm i} \lambda \Delta K | I_{\rm f} K_{\rm f} \rangle + ... \right], \quad (2.3b)\end{aligned}$$

where the quantity $C_{\rm if}$ is defined as

$$C_{\rm if} \equiv \begin{cases} 1 & \text{if } K_{\rm i} = K_{\rm f} = 0 \text{ or } K_{\rm i} \neq 0,\ K_{\rm f} \neq 0, \\ \sqrt{2} & \text{if } K_{\rm i} = 0,\ K_{\rm f} \neq 0 \text{ or } K_{\rm i} \neq 0,\ K_{\rm f} = 0, \end{cases} \quad (2.4)$$

The explicit form of the lowest-order-correction terms is not shown in Eq.(2.3b) but will be discussed later. Thus the GIR is completely determined by the intrinsic matrix elements, $(\mathrm{f}|\widehat{m}^{(0)}_{\lambda\Delta K}|\mathrm{i})$, $(\mathrm{f}|\widehat{m}^{(\pm1)}_{\lambda\Delta K\pm1}|\mathrm{i})$, etc.. Needless to say, the operator $\widehat{m}^{(\pm1)}_{\lambda\Delta K\pm1}$ with $|\Delta K \pm 1| > \lambda$ is understood to be vanishing, and furthermore, the initial and final states, $|\mathrm{i})$ and $|\mathrm{f})$, must have good K-quantum numbers ($K \geq 0$). Here and hereafter we use round brackets to denote the states and the intrinsic matrix elements in the unified model.

By contrast, the semi-microscopic cranking formalism of Ref. 4) leads to a simpler form for the reduced transition amplitude, Eq.(2.3b), if it is combined with the $1/I$-expansion

* The intrinsic operator $\widehat{m}^{(n)}_{\lambda\nu}$ corresponds to the moment $m_{\nu-n,\nu}$ of Ref.2).

technique (valid in the high-spin limit),

$$\mathcal{T}(\lambda : \mathrm{i} \to \mathrm{f}) \approx \langle \mathrm{f}|\widetilde{Q}_{\lambda\mu=\Delta I}|\mathrm{i}\rangle, \tag{2.5}$$

where $\Delta I \equiv I_\mathrm{f} - I_\mathrm{i}$ and $\widetilde{Q}_{\lambda\mu}$ is the transition operator defined with respect to the rotation (cranking) axis and the matrix element $\langle \mathrm{f}|\widetilde{Q}_{\lambda\mu}|\mathrm{i}\rangle$ is microscopically calculated by the usual cranking prescription at finite rotational frequency ω_rot (we use $\hbar = 1$ unit in this paper). The initial and final states in the cranking model, $|\mathrm{i}\rangle$ and $|\mathrm{f}\rangle$, are prepared in such a way as to guarantee a good signature-quantum number, which is consistent with using the good-signature operator $\widetilde{Q}_{\lambda\mu=\Delta I}$.

The essential idea is to consider the limit, $\omega_\mathrm{rot} \to 0$, which is opposite to that normally used in the cranking formalism and to relate the quantities appearing in the cranking calculations to the intrinsic matrix elements in the unified model in Eq.(2.3). At first sight this may seem impossible, but we will show that it can be done for arbitrary types of transitions due to a remarkable correspondence between (2.3) and (2.5) (it has already been done for the $E2$ transition from the γ-vibrational band to the ground-state band in Ref. 5)).

The first step in obtaining the correspondence is to correct the difference of the basis states used in the two models: K is a good quantum number in the unified model, while signature is good in the cranking prescription. The states with $K \neq 0$ are degenerate at $\omega_\mathrm{rot} = 0$, and roughly speaking, a pair of degenerate signature-conjugate states, say, $|\mathrm{i}\rangle$ and $|\bar{\mathrm{i}}\rangle$, form a strongly-coupled $\Delta I = 1$ rotational band corresponding to $|\mathrm{i})$. Thus, the starting point, Eq.(2.5) should be modified:

$$\mathcal{T}(\lambda : \mathrm{i} \to \mathrm{f}) \approx C_\mathrm{if}\, \langle\langle \mathrm{f}|\widetilde{Q}_{\lambda\mu=\Delta I}|\mathrm{i}\rangle\rangle, \tag{2.6}$$

where the states $|\mathrm{i(f)}\rangle\rangle$ are constructed as a linear combination of $|\mathrm{i(f)}\rangle$ and it's signature-conjugate states $|\bar{\mathrm{i}}(\bar{\mathrm{f}})\rangle$ so as to make K a good quantum number in the limit $\omega_\mathrm{rot} = 0$. Since $|\mathrm{i}\rangle$ and $|\bar{\mathrm{i}}\rangle$ are independent, the relative phase between them is free within the cranking formalism. We fix the phase in this paper by,

$$K_\mathrm{i} = \langle\bar{\mathrm{i}}|J_z|\mathrm{i}\rangle = \langle \mathrm{i}|J_z|\bar{\mathrm{i}}\rangle \geq 0, \qquad \text{at} \quad \omega_\mathrm{rot} = 0. \tag{2.7a}$$

Then the $|\mathrm{i}\rangle\!\rangle$ is explicitly defined by

$$\begin{cases} |\mathrm{i}\rangle\!\rangle = |\mathrm{i}\rangle & \text{for } K_\mathrm{i} = 0, \\ |\mathrm{i}\rangle\!\rangle = \dfrac{1}{\sqrt{2}}\big(|\mathrm{i}\rangle + |\bar{\mathrm{i}}\rangle\big) & \text{for } K_\mathrm{i} > 0, \\ |-\mathrm{i}\rangle\!\rangle = \dfrac{1}{\sqrt{2}}\big(|\mathrm{i}\rangle - |\bar{\mathrm{i}}\rangle\big) & \text{for } K_\mathrm{i} < 0, \end{cases} \tag{2.7b}$$

which has definite K_i values in the $\omega_\mathrm{rot} = 0$ limit (the same for $|\mathrm{f}\rangle\!\rangle$). We call a set of states defined in this way a "K-good" representation, though they do not have good K quantum numbers at finite ω_rot. In the following, we use $|\mathrm{i}\rangle\!\rangle$ ($|\mathrm{f}\rangle\!\rangle$) for states with non-negative K-values and $|-\mathrm{i}\rangle\!\rangle$ ($|-\mathrm{f}\rangle\!\rangle$) for those with negative K. The coefficient C_if in Eq.(2.6), which is defined by Eq.(2.4) through the K_i and K_f values at $\omega_\mathrm{rot} = 0$, is introduced in order to guarantee that Eq.(2.6) recovers the original Eq.(2.5) in the high-spin limit.

The next step is to decompose the multipole transition operators $\widetilde{Q}_{\lambda\mu}$ in Eq.(2.6), which are defined with respect to the rotation axis (x-axis), into those defined with respect to the symmetry axis (z-axis), $Q_{\lambda\nu}$:

$$\widetilde{Q}_{\lambda\mu} = \sum_\nu \mathcal{D}^\lambda_{\mu\nu}\Big(-\frac{\pi}{2}, -\frac{\pi}{2}, 0\Big)\, Q_{\lambda\nu} = i^{-\mu} \sum_\nu d^\lambda_{\mu\nu}\Big(-\frac{\pi}{2}\Big)\, Q_{\lambda\nu}, \tag{2.8}$$

where we followed Ref. 2) for definitions of the $\mathcal{D}$ and d functions. The original GIR was obtained by expanding the matrix elements with respect to the angular momentum. Correspondingly, the matrix elements in the cranking prescription are expanded with respect to the rotational frequency:

$$\langle\!\langle \mathrm{f}|Q_{\lambda\nu}|\mathrm{i}\rangle\!\rangle = \begin{cases} O(1) & \text{for} \quad \nu = \Delta K, \\ O(\omega_\mathrm{rot}) & \text{for} \quad \nu = \Delta K \pm 1, \\ O(\omega^2_\mathrm{rot}) & \text{otherwise}, \end{cases} \tag{2.9}$$

because the cranking term mixes the states with $\Delta K = \pm 1$. Using Eqs.(2.8) and (2.9), we obtain within the lowest-order corrections with respect to ω_rot,

$$\begin{aligned} \langle\!\langle \mathrm{f}|\widetilde{Q}_{\lambda\mu=\Delta I}|\mathrm{i}\rangle\!\rangle = i^{-\Delta I} \Bigg(& \Big[\langle\!\langle \mathrm{f}|Q_{\lambda\Delta K}|\mathrm{i}\rangle\!\rangle\Big]_0 d^\lambda_{\Delta I \Delta K}\Big(-\frac{\pi}{2}\Big) \\ & + \Big[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda\Delta K+1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\Big]_0 \omega_\mathrm{rot}\, d^\lambda_{\Delta I \Delta K+1}\Big(-\frac{\pi}{2}\Big) \\ & + \Big[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda\Delta K-1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\Big]_0 \omega_\mathrm{rot}\, d^\lambda_{\Delta I \Delta K-1}\Big(-\frac{\pi}{2}\Big) \Bigg). \end{aligned} \tag{2.10}$$

Here $[*]_0$ means that the expression is evaluated by taking the limit $\omega_{\rm rot} \to 0$.

The key to obtain the correspondence is the d function, which has an asymptotic form,[6)]

$$d^{\lambda}_{\Delta I \Delta K}(-\theta) \approx \langle I_{\rm i} K_{\rm i} \lambda \Delta K | I_{\rm f} K_{\rm f} \rangle, \tag{2.11a}$$

with

$$\Delta I \equiv I_{\rm f} - I_{\rm i},\ \Delta K \equiv K_{\rm f} - K_{\rm i}, \quad \cos\theta \approx K_{\rm i}/I_{\rm i} \approx K_{\rm f}/I_{\rm f}, \tag{2.11b}$$

which is valid for integer λ and for $\Delta I, \Delta K \ll I_{\rm i}, I_{\rm f}$. The cranking prescription, Eq.(2.6) with Eq.(2.8), takes the high-spin limit $K \ll I$, namely $\theta \approx \frac{\pi}{2}$. Since we are considering the opposite limit ($\theta < \frac{\pi}{2}$), it is natural to replace the d function of the leading-order term in Eq.(2.10) by the Clebsch-Gordan coefficient (2.11a). Then the leading-order matrix element in the cranking prescription (2.6) turns out to have the same structure as the one in the unified model, Eq.(2.3). An alternative interpretation of this replacement is that the d function $d^{\lambda}_{\Delta I \Delta K}$ in Eq.(2.10) corresponds to the reduced matrix element of the $\mathcal{D}$ operator in Eq.(2.3) with respect to the rotor part of unsymmetrized wave functions in Eq.(2.1): i.e.

$$d^{\lambda}_{\Delta I \Delta K}\Big(-\frac{\pi}{2}\Big) \ \leftrightarrow\ \Big[\frac{2I_{\rm f}+1}{2I_{\rm i}+1}\Big]^{\frac{1}{2}} \ \frac{\langle K_{\rm f} I_{\rm f} M_{\rm f} | \mathcal{D}^{\lambda}_{\mu \Delta K} | K_{\rm i} I_{\rm i} M_{\rm i} \rangle_{\rm unsym}}{\langle I_{\rm i} M_{\rm f} \lambda \mu | I_{\rm f} M_{\rm f} \rangle}. \tag{2.12a}$$

Note that this replacement may be regarded as a kind of "quantization" incorporating the full quantum-mechanical effects of angular-momentum algebra present in the GIR but neglected in the semiclassical cranking model.

With the close relationship between the $\omega_{\rm rot}$ expansion in the cranking calculation and the I expansion in the GIR, we may introduce a "quantization" for the higher-order term: i.e.

$$\omega_{\rm rot}\, d^{\lambda}_{\Delta I \Delta K \pm 1}\Big(-\frac{\pi}{2}\Big) \ \leftrightarrow\ \Big[\frac{2I_{\rm f}+1}{2I_{\rm i}+1}\Big]^{\frac{1}{2}} \ \frac{\langle K_{\rm f} I_{\rm f} M_{\rm f} | \ \dfrac{1}{\mathcal{J}} \dfrac{1}{2} \{ I_{\pm}, \mathcal{D}^{\lambda}_{\mu, \Delta K \pm 1} \} \ | K_{\rm i} I_{\rm i} M_{\rm i} \rangle_{\rm unsym}}{\langle I_{\rm i} M_{\rm f} \lambda \mu | I_{\rm f} M_{\rm f} \rangle}, \tag{2.12b}$$

where $\mathcal{J}$ is the moment of inertia of the band under consideration at $\omega_{\rm rot} = 0$. Noting that $\omega_{\rm rot} \approx I_x/\mathcal{J}$, the "quantization" of Eq.(2.12b), i.e. the appearance of $\dfrac{1}{\mathcal{J}} \dfrac{1}{2} \{ I_{\pm}, \mathcal{D}^{\lambda}_{\mu\nu} \}$ on the right hand side, is natural, though the ordering between $I_{\pm}$ and $\mathcal{D}^{\lambda}_{\mu\nu}$ is not trivial. As in

any kind of quantization procedures, the ordering of the operators cannot be determined from classical considerations. We have found that the symmetrized ordering as given in Eq.(2.12b) is the most appropriate one and it is discussed further in Appendix A. Therefore we take this ordering throughout the present paper. Concerning the moment of inertia, there is ambiguity because its values are generally different for the initial and final states. However, since the difference is small in most cases, we may say that

$$\mathcal{J} \equiv \frac{1}{2}\Big[\frac{d\langle\langle \mathrm{i}|J_x|\mathrm{i}\rangle\rangle}{d\omega_{\rm rot}} + \frac{d\langle\langle \mathrm{f}|J_x|\mathrm{f}\rangle\rangle}{d\omega_{\rm rot}}\Big]_0 \approx \Big[\frac{d\langle\langle \mathrm{i}|J_x|\mathrm{i}\rangle\rangle}{d\omega_{\rm rot}}\Big]_0 \approx \Big[\frac{d\langle\langle \mathrm{f}|J_x|\mathrm{f}\rangle\rangle}{d\omega_{\rm rot}}\Big]_0. \tag{2.13}$$

Using these quantization conditions, Eq.(2.12), the correspondence between Eq.(2.3) and Eq.(2.6) with (2.10) is apparent and the cranking procedure gives the intrinsic matrix elements in the unified model within the lowest-order corrections by

$$(\mathrm{f}|\widehat{m}^{(0)}_{\lambda\Delta K}|\mathrm{i}) = \Big[\langle\langle \mathrm{f}|Q_{\lambda\Delta K}|\mathrm{i}\rangle\rangle\Big]_0, \tag{2.14a}$$

$$(\mathrm{f}|\widehat{m}^{(\pm 1)}_{\lambda\Delta K\pm 1}|\mathrm{i}) = \frac{1}{\mathcal{J}}\Big[\frac{d\langle\langle \mathrm{f}|Q_{\lambda\Delta K\pm 1}|\mathrm{i}\rangle\rangle}{d\omega_{\rm rot}}\Big]_0. \tag{2.14b}$$

Here and hereafter we omit the overall phase $i^{-\Delta I}$ which comes from Eq.(2.10).

Although the procedure is, in principle, applicable to any order in angular momentum, it becomes much more difficult to deduce the general correspondence because the number of possible orderings increases. The only exception is the K-forbidden transitions.[2] For the transition with the order of K-forbiddenness, $n = |\Delta K| - \lambda$ (≥ 0), the operator has the form

$$\begin{aligned}\mathcal{M}(\lambda\mu)_{\Delta K=\pm(n+\lambda)} =& \widehat{m}^{(\mp n)}_{\lambda,\pm\lambda}\, I^n_{\mp}\, \mathcal{D}^{\lambda}_{\mu,\pm\lambda} + \widehat{m}^{(\mp(n+1))}_{\lambda,\pm(\lambda-1)}\, \frac{1}{2}\{I^{n+1}_{\mp}, \mathcal{D}^{\lambda}_{\mu,\pm(\lambda-1)}\} \\ &+ \mathcal{R}-\text{conj.}.\end{aligned} \tag{2.15}$$

In this case, the $\omega_{\rm rot}$-expansion, Eq.(2.9), gives the leading-order matrix elements, $\langle\langle \mathrm{f}|Q_{\lambda,\pm\lambda}|\mathrm{i}\rangle\rangle = O(\omega^n_{\rm rot})$, and the first-order correction, $\langle\langle \mathrm{f}|Q_{\lambda,\pm(\lambda-1)}|\mathrm{i}\rangle\rangle = O(\omega^{n+1}_{\rm rot})$. Then the "quantization" with the symmetrized ordering as in Eqs.(2.12) leads to the correspondence

$$(\mathrm{f}|\widehat{m}^{(\mp n)}_{\lambda,\pm\lambda}|\mathrm{i}) = \frac{1}{n!\mathcal{J}^n}\Big[\frac{d^n\langle\langle \mathrm{f}|Q_{\lambda,\pm\lambda}|\mathrm{i}\rangle\rangle}{d\omega^n_{\rm rot}}\Big]_0, \tag{2.16a}$$

$$(\mathrm{f}|\widehat{m}^{(\mp(n+1))}_{\lambda,\pm(\lambda-1)}|\mathrm{i}) = \frac{1}{(n+1)!\mathcal{J}^{n+1}}\Big[\frac{d^{n+1}\langle\langle \mathrm{f}|Q_{\lambda,\pm(\lambda-1)}|\mathrm{i}\rangle\rangle}{d\omega^{n+1}_{\rm rot}}\Big]_0. \tag{2.16b}$$

Note that there exists a one-to-one correspondence between the unified-model intrinsic state $|\mathrm{i})$ and the cranking-model state $|\mathrm{i}\rangle\!\rangle$ because of the introduction of the K-good representation (2.7) at $\omega_{\rm rot}=0$. From Eqs.(2.14) and (2.16), it may be generally written

$$(\mathrm{f}|\widehat{m}^{(n)}_{\lambda\nu}|\mathrm{i}) \equiv \frac{1}{|n|!\mathcal{J}^{|n|}}\Big[\frac{d^{|n|}\langle\!\langle \mathrm{f}|Q_{\lambda\nu}|\mathrm{i}\rangle\!\rangle}{d\omega^{|n|}_{\rm rot}}\Big]_0, \quad \text{for } \Delta K=\nu-n. \tag{2.17}$$

With this simple prescription, we are able to calculate microscopically the intrinsic matrix elements in the unified model by using the cranking model.

Although they are not considered explicitly in the following examples, it is worthwhile mentioning that signature-dependent contributions (decoupling terms) can be evaluated in the same way as in the original treatment of the GIR; namely the intrinsic matrix elements are

$$(\mathrm{f}|\widehat{m}^{(n)}_{\lambda\nu}\mathcal{R}_i^{-1}|\mathrm{i}) = (-)^{K_{\rm i}}\,(\mathrm{f}|\widehat{m}^{(n)}_{\lambda\nu}\mathcal{R}_x^{-1}|\mathrm{i}) = (-)^{\alpha_{\rm i}+K_{\rm i}}\frac{1}{|n|!\mathcal{J}^{|n|}}\Big[\frac{d^{|n|}\langle\!\langle \mathrm{f}|Q_{\lambda\nu}|-\mathrm{i}\rangle\!\rangle}{d\omega^{|n|}_{\rm rot}}\Big]_0, \tag{2.18}$$

where the intrinsic $\mathcal{R}$-conjugation operator[2] is $\mathcal{R}_i=\exp(-i\pi J_y)$, while the signature-conjugation operator is $\mathcal{R}_x=\exp(-i\pi J_x)$, and $\alpha_{\rm i}$ is the signature-quantum number of the cranking state $|\mathrm{i}\rangle$, i.e. $\mathcal{R}_x|\mathrm{i}\rangle=(-1)^{-\alpha_{\rm i}}|\mathrm{i}\rangle$.

In the unified model,[2] it is straightforward to obtain the GIR from the wave function (2.1) and the transition operator (2.2). The main result of the present paper is the description of a microscopic method to calculate the intrinsic matrix elements in the $\omega_{\rm rot}\to 0$ limit of the cranking model.

§3 Some Applications

In the following, we take concrete examples of various types of transitions and show explicitly the formula to calculate the GIR in terms of the signature-good matrix elements of the cranking model. We assume the usual phase convention[7] for the matrix elements of electromagnetic transition operators.

Since the eigenstates in cranking calculations have good signature, it is useful to introduce the signature-classified-transition operators,

$$Q^{(\pm)}_{\lambda K} \equiv \frac{1}{\sqrt{2(1+\delta_{K0})}}\big(Q_{\lambda K}\pm(-)^{\lambda}Q_{\lambda,-K}\big), \qquad K\geq 0, \tag{3.1a}$$

where the sign $+(-)$ means that the operator transfers the signature by $r = +1(-1)$, or $\alpha = 0\,(1)$. Conversely,

$$Q_{\lambda,\pm K} = \frac{1}{\sqrt{2-\delta_{K0}}} \begin{cases} \big(Q^{(+)}_{\lambda K} \pm Q^{(-)}_{\lambda K}\big), & \text{for} \quad \lambda = \text{even}, \\ \big(Q^{(-)}_{\lambda K} \pm Q^{(+)}_{\lambda K}\big), & \text{for} \quad \lambda = \text{odd}. \end{cases} \tag{3.1b}$$

Only one signature component exists for the $K = 0$ operator, namely the one with $(+)$ for λ=even and $(-)$ for λ=odd. Using Eqs.(2.7) and (3.1), one can easily convert the matrix elements in the K-good states, $\langle\langle \mathrm{f}|Q_{\lambda K}|\mathrm{i}\rangle\rangle$, to those in the signature-good states, e.g. $\langle \mathrm{f}|Q^{(+)}_{\lambda K}|\mathrm{i}\rangle$ and $\langle \bar{\mathrm{f}}|Q^{(-)}_{\lambda K}|\mathrm{i}\rangle$ etc., which are used in the cranking calculations.

3-1. Intraband M1 and E2 transitions

Let us first consider the simplest example of the in-band $M1$ transition. In this case the initial and final states are the same, $|\mathrm{i}\rangle = |\mathrm{f}\rangle$, $\Delta K = 0$ and $K_\mathrm{i} = K_\mathrm{f} \equiv K \neq 0$. We denote these states as $|K\rangle$ for simplicity. The magnetic dipole operator $\vec{\mu}$ is classified as

$$\mu^{(-)}_{10} = \mu_z, \;\; \mu^{(+)}_{11} = -\mu_x, \;\; \text{and} \;\; \mu^{(-)}_{11} = -i\mu_y.$$

Then the leading-order matrix element (2.14a) is

$$(\mathrm{f}|\widehat{m}^{(0)}_{10}|\mathrm{i}) = \Big[\langle\langle K|\mu_z|K\rangle\rangle\Big]_0 = g_K K, \quad g_K \equiv \Big[\frac{\langle \bar{K}|\mu_z|K\rangle}{\langle \bar{K}|J_z|K\rangle}\Big]_0, \tag{3.2a}$$

where $\langle \bar{K}|\mu_z|K\rangle = \langle K|\mu_z|\bar{K}\rangle$ and the phase conventions (2.7) are used. In the case of $M1$ transitions, the first-order-correction term is an important contribution of the same order as the leading term.[2] In fact, by using Eq.(2.14b) the first-order matrix element is

$$\begin{aligned} (\mathrm{f}|\widehat{m}^{(\pm 1)}_{1,\pm 1}|\mathrm{i}) &= \mp\frac{1}{\sqrt{2}\mathcal{J}}\Big[\frac{d\langle\langle K|\mu_x|K\rangle\rangle}{d\omega_\mathrm{rot}}\Big]_0 = \mp\frac{1}{\sqrt{2}}\, g_\mathrm{R}, \\ g_\mathrm{R} &\equiv \Big[\Big(\frac{d\langle K|\mu_x|K\rangle}{d\omega_\mathrm{rot}} + \frac{d\langle \bar{K}|\mu_x|\bar{K}\rangle}{d\omega_\mathrm{rot}}\Big)\Big/\Big(\frac{d\langle K|J_x|K\rangle}{d\omega_\mathrm{rot}} + \frac{d\langle \bar{K}|J_x|\bar{K}\rangle}{d\omega_\mathrm{rot}}\Big)\Big]_0, \end{aligned} \tag{3.2b}$$

where we have used $\langle\langle K|i\mu_y|K\rangle\rangle = 0$, and $|K\rangle = |\bar{K}\rangle$ for $K = 0$ bands. Then, by using the identity,

$$\frac{1}{\sqrt{2}}\Big(-\frac{1}{2}\big\{I_+, \mathcal{D}^1_{\mu,+1}\big\} + \frac{1}{2}\big\{I_-, \mathcal{D}^1_{\mu,-1}\big\}\Big) = I^{(\mathrm{lab})}_\mu - I_0\mathcal{D}^1_{\mu 0}, \quad I^{(\mathrm{lab})}_\mu \equiv \sum_\nu \mathcal{D}^1_{\mu\nu} I_\nu,$$

and the fact that I_0 gives K when operated, it is easy to see that

$$\mathcal{T}(M1:\mathrm{i}\to\mathrm{f})_{\mathrm{in}} = g_{\mathrm{R}}\sqrt{I_{\mathrm{f}}(I_{\mathrm{f}}+1)}\,\delta_{I_{\mathrm{i}}I_{\mathrm{f}}} + \langle I_{\mathrm{i}}K10|I_{\mathrm{f}}K\rangle\,(g_K - g_{\mathrm{R}})K. \tag{3.3}$$

Thus the well-known formula for the $M1$ matrix elements is derived, where the g factors, g_K as well as g_{R}, can be calculated from the microscopic cranking formalism at the infinitesimal rotational frequency point.

The second simple example is the in-band $E2$ transitions. In this case the leading-order matrix element (2.14a) becomes the quadrupole moment:

$$\begin{aligned}(\mathrm{f}|\widehat{m}^{(0)}_{20}|\mathrm{i}) &= \Big[\langle\!\langle K|Q^{(+)}_{20}|K\rangle\!\rangle\Big]_0 = Q_0,\\ Q_0 &\equiv \Big[\frac{1}{2}\Big(\langle K|Q^{(+)}_{20}|K\rangle + \langle\bar{K}|Q^{(+)}_{20}|\bar{K}\rangle\Big)\Big]_0 = \Big[\langle K|Q^{(+)}_{20}|K\rangle\Big]_0.\end{aligned} \tag{3.4a}$$

The lowest-order correction term, which is linear in angular momentum, is non-vanishing if $K \neq 0$; by using Eq.(2.14b),

$$(\mathrm{f}|\widehat{m}^{(\pm1)}_{2,\pm1}|\mathrm{i}) = \pm Q'_1, \quad Q'_1 \equiv \frac{1}{\sqrt{2}\mathcal{J}}\Big[\frac{d\langle\bar{K}|Q^{(-)}_{21}|K\rangle}{d\omega_{\mathrm{rot}}}\Big]_0(1-\delta_{K0}). \tag{3.4b}$$

Here we have used $\langle\!\langle K|Q^{(+)}_{21}|K\rangle\!\rangle = 0$ and $\langle\bar{K}|Q^{(-)}_{21}|K\rangle = \langle K|Q^{(-)}_{21}|\bar{K}\rangle$. Then the GIR is written as

$$\begin{aligned}\mathcal{T}(E2:\mathrm{i}\to\mathrm{f})_{\mathrm{in}} &= \langle I_{\mathrm{i}}K20|I_{\mathrm{f}}K\rangle\,Q_0\\ &\quad + \Big(\langle I_{\mathrm{i}}K21|I_{\mathrm{f}}K+1\rangle\sqrt{(I_{\mathrm{f}}-K)(I_{\mathrm{f}}+K+1)}\\ &\quad - \langle I_{\mathrm{i}}K2-1|I_{\mathrm{f}}K-1\rangle\sqrt{(I_{\mathrm{f}}+K)(I_{\mathrm{f}}-K+1)}\Big)\,Q'_1,\end{aligned} \tag{3.5}$$

where the lowest-correction term can be written in various alternative ways:[2)] We have used the identity,

$$\frac{1}{2}\Big(\big\{I_+,\mathcal{D}^2_{\mu,+1}\big\} - \big\{I_-,\mathcal{D}^2_{\mu,-1}\big\}\Big) = I_+\,\mathcal{D}^2_{\mu,+1} - I_-\,\mathcal{D}^2_{\mu,-1}.$$

The first and second terms in the right hand side of Eq.(3.5) come from the matrix elements with signature $\alpha = 0$ and $\alpha = 1$, respectively. It is interesting to verify the selection rule, $\Delta I = \alpha\ (mod\,2)$, which is valid in the high-spin cranking formalism. In fact,

the ratio of these terms takes the asymptotic value,

$$\frac{\alpha=0 \text{ term}}{\alpha=1 \text{ term}} \approx \frac{Q_0}{IQ_1'} \times \begin{cases} \dfrac{1}{2\sqrt{6}}(K/I)^{-1} & \text{for} \quad \Delta I = \pm 2, \\ -\dfrac{\sqrt{6}}{2}(K/I) & \text{for} \quad \Delta I = \pm 1, \\ \dfrac{\sqrt{6}}{4}(K/I)^{-1} & \text{for} \quad \Delta I = 0, \end{cases}$$

in the limit of $I_{\rm f} \approx I_{\rm i} \approx I \gg K \gg 1$. Thus, the correspondence between signature and angular momentum in the cranking formalism is recovered at the high-spin limit.

For the one-quasiparticle states, Q_1' is a single-particle matrix element, while Q_0 is the collective matrix element. Therefore the Q_1' is typically two to three orders of magnitude smaller than Q_0 and gives a negligible contribution in our formalism. This is in agreement with experimental observations.[2)]

3-2. K-allowed transitions

It is straightforward, but tedious, to derive the GIR for arbitrary transitions in the general case because the number of non-zero matrix element increases due to many possible ways of transferring the K-quantum number, and because of the signature distinction. However, a rather simple formula can be obtained for the general λ-pole transitions if we neglect the signature-dependent contributions and keep the matrix elements in the K-good representation as in Eq.(2.17). The GIR can then be written in different ways depending on which Clebsch-Gordan coefficients are included. A simple form of the GIR, Eq.(4-98) of Ref.2), was derived assuming $\Delta K = K_{\rm f} - K_{\rm i} \geq \lambda$. In this section, we present the formula for arbitrary K-allowed transitions with $|\Delta K| \leq \lambda$.

Since the signature-dependent terms are neglected there exist two first-order-correction terms associated with the rotor part operator, $\frac{1}{2}\{I_+, \mathcal{D}^\lambda_{\mu,\Delta K+1}\}$ and $\frac{1}{2}\{I_-, \mathcal{D}^\lambda_{\mu,\Delta K-1}\}$ as is seen in Eq.(2.3a). Let us call them simply I_+ and I_- terms, respectively. It is always possible to eliminate one of them by using a general identity between the angular momenta and the $\mathcal{D}$ function,[2)]

$$[(\lambda-\nu)(\lambda+\nu+1)]^{\frac{1}{2}} \frac{1}{2}\{I_+, \mathcal{D}^\lambda_{\mu,\nu+1}\} + [(\lambda+\nu)(\lambda-\nu+1)]^{\frac{1}{2}} \frac{1}{2}\{I_-, \mathcal{D}^\lambda_{\mu,\nu-1}\} = [\boldsymbol{I}^2, \mathcal{D}^\lambda_{\mu\nu}] - \nu\{I_0, \mathcal{D}^\lambda_{\mu\nu}\}. \tag{3.6}$$

For $-\lambda \leq \nu = \Delta K \leq 0$ ($\lambda \geq \nu = \Delta K \geq 0$) we can eliminate the I_+ (I_-) term so as to obtain a simple form of GIR. In order to further simplify the relation we apply the identity,

$$\frac{1}{2}\{I_\pm, \mathcal{D}^\lambda_{\mu,\nu\pm1}\} = I_\pm \mathcal{D}^\lambda_{\mu,\nu\pm1} - \frac{1}{2}\sqrt{(\lambda \mp \nu)(\lambda \pm \nu + 1)}\, \mathcal{D}^\lambda_{\mu\nu}, \tag{3.7}$$

to the remaining I_- (I_+) term. Then the GIR for the λ-pole transitions is written as

$$\begin{aligned}\mathcal{T}(\lambda : \mathrm{i} \to \mathrm{f})_{\Delta K} =& \langle I_\mathrm{i} K_\mathrm{i} \lambda \Delta K | I_\mathrm{f} K_\mathrm{f}\rangle\, Q_t \left(1 + q\left[I_\mathrm{f}(I_\mathrm{f}+1) - I_\mathrm{i}(I_\mathrm{i}+1)\right]\right) \\ &+ \sqrt{(I_\mathrm{f} \pm K_\mathrm{f})(I_\mathrm{f} \mp K_\mathrm{f} + 1)}\, \langle I_\mathrm{i} K_\mathrm{i} \lambda\, (\Delta K \mp 1) | I_\mathrm{f}\, (K_\mathrm{f} \mp 1)\rangle\, Q'_t, \quad (\Delta K \lessgtr 0), \end{aligned} \tag{3.8}$$

where the upper (lower) sign is for $\Delta K \leq 0$ ($\Delta K \geq 0$), and the intrinsic parameters Q_t, Q'_t and q are calculated in the $\omega_\mathrm{rot} \to 0$ limit of the cranking formalism by means of Eq.(2.17);

$$\begin{aligned} Q_t =& C_\mathrm{if}\left\{ \left[\langle\!\langle \mathrm{f}|Q_{\lambda\Delta K}|\mathrm{i}\rangle\!\rangle\right]_0 - \frac{\sqrt{(\lambda \pm \Delta K)(\lambda \mp \Delta K + 1)}}{2\mathcal{J}} \left[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda,\Delta K\mp1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\right]_0 \right. \\ &\left. + \frac{(\lambda \pm \Delta K)(\lambda \mp \Delta K + 1) - 2\Delta K(K_\mathrm{i} + K_\mathrm{f})}{\sqrt{(\lambda \mp \Delta K)(\lambda \pm \Delta K + 1)}}\, \frac{1}{2\mathcal{J}} \left[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda,\Delta K\pm1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\right]_0 \right\}, \\ Q'_t =& C_\mathrm{if}\left\{ \frac{1}{\mathcal{J}}\left[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda,\Delta K\mp1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\right]_0 - \sqrt{\frac{(\lambda \pm \Delta K)(\lambda \mp \Delta K + 1)}{(\lambda \mp \Delta K)(\lambda \pm \Delta K + 1)}}\, \frac{1}{\mathcal{J}} \left[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda,\Delta K\pm1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\right]_0 \right\}, \\ q =& C_\mathrm{if}\, \frac{1}{\sqrt{(\lambda \mp \Delta K)(\lambda \pm \Delta K + 1)}} \frac{1}{\mathcal{J}} \left[\frac{d\langle\!\langle \mathrm{f}|Q_{\lambda,\Delta K\pm1}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}}\right]_0 / Q_t, \quad (\Delta K \lessgtr 0). \end{aligned} \tag{3.9}$$

Note that Q'_t vanishes when $\Delta K = \mp\lambda$ and the simple GIR[2)] is recovered. By using Eqs.(3.6) and (3.7) it is easy to check that the formula for $\Delta K \leq 0$ and $\Delta K \geq 0$ give identical results when $\Delta K = 0$. The results of §3-1 can be derived from the general formula above, though their appearance is different at first sight. In Eq.(3.8), Q_t is, roughly speaking, the leading-order term and the Q'_t and q terms give the lowest-order corrections in the sense of I expansion. It should be mentioned that the higher-order terms in the main amplitude Q_t, Eq.(3.9), come from the "contraction" due to the specific ordering expressed in Eq.(2.12), which should be small compared to the main term, but they do give important contributions.

3-3. K-forbidden transitions

The simple GIR, Eq.(4-98) in Ref.2), can be obtained for the K-forbidden transitions. Again, we consider both the $\Delta K < 0$ and $\Delta K > 0$ cases. Using the order of

K-forbiddenness, $n = |\Delta K| - \lambda > 0$, and applying Eq.(2.17) to Eqs.(2.15) and (2.16), we obtain

$$\mathcal{T}(\lambda : \mathrm{i} \to \mathrm{f})_{\Delta K = \mp(n+\lambda)} = \sqrt{\frac{(I_\mathrm{f} \mp K_\mathrm{f})!\,(I_\mathrm{f} \pm K_\mathrm{f} + n)!}{(I_\mathrm{f} \mp K_\mathrm{f} - n)!\,(I_\mathrm{f} \pm K_\mathrm{f})!}}\,\langle I_\mathrm{i} K_\mathrm{i} \lambda\,(\mp\lambda) | I_\mathrm{f}\,(K_\mathrm{f} \pm n)\rangle\, Q_t \times \big(1 + q\,[I_\mathrm{f}(I_\mathrm{f}+1) - I_\mathrm{i}(I_\mathrm{i}+1)]\,\big), \quad (\Delta K \lessgtr 0), \tag{3.10}$$

where

$$Q_t = C_\mathrm{if}\left\{ \frac{1}{n!\mathcal{J}^n}\Big[\frac{d^n \langle\!\langle \mathrm{f}|Q_{\lambda,\mp\lambda}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}^n}\Big]_0 \pm \sqrt{\frac{\lambda}{2}}\,\frac{(K_\mathrm{i} + K_\mathrm{f})}{(n+1)!\mathcal{J}^{n+1}}\Big[\frac{d^{n+1}\langle\!\langle \mathrm{f}|Q_{\lambda,\mp(\lambda-1)}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}^{n+1}}\Big]_0 \right\},$$

$$q = C_\mathrm{if}\frac{1}{\sqrt{2\lambda}\,(n+1)!\mathcal{J}^{n+1}}\Big[\frac{d^{n+1}\langle\!\langle \mathrm{f}|Q_{\lambda,\mp(\lambda-1)}|\mathrm{i}\rangle\!\rangle}{d\omega_\mathrm{rot}^{n+1}}\Big]_0 / Q_t, \quad (\Delta K \lessgtr 0). \tag{3.11}$$

Once the initial and final states are specified, the intrinsic parameters (3.11) can be calculated in the cranking model. This formula may find application to analyzing the direct decays of the high-K isomer states to the ground-state band, which have been systematically observed recently. For two-quasiparticle high-K isomers, both $M1$ and $E2$ matrix elements are straightforward to evaluate within the independent quasiparticle approximation. Some correlations between the quasiparticles will be necessary in order to obtain non-zero amplitudes for three- or four-quasiparticle isomers.

3-4. Vibrational transitions to the ground-state band of even-even nuclei

One of the important applications of the GIR is to transitions from collective vibrational bands to the ground-state band. Collective vibrational motion built on the ground band can be consistently described within the cranking model by employing the random-phase approximation (RPA) in the rotating frame,[4,8–12] which we will call the simple RPA method hereafter. In this subsection, we discuss the GIR for vibrational transitions in even-even nuclei with the simple RPA method.

Since $K_\mathrm{f} = 0$ for the ground-state band of even-even nuclei, we have $K_\mathrm{i} = K$ and $\Delta K = -K \leq 0$ for the λ-pole transitions from the λ'-pole vibrational mode with definite K ($0 \leq K \leq \lambda'$) in the $\omega_\mathrm{rot} = 0$ limit. Moreover, in the simple RPA method, the initial and final states are simply related by $|\mathrm{i}\rangle = X_K^{(\pm)\dagger}|\mathrm{f}\rangle$, where $|\mathrm{f}\rangle = |0\rangle$ denote the ground state rotational band and $X_K^{(\pm)\dagger}$ is the creation operator of the vibrational mode with the

signature $r = \pm 1$ and the K-quantum number K at $\omega_{\rm rot} = 0$. The RPA transition matrix elements between the one-phonon states and the vacuum state at an arbitrary rotational frequency are defined by

$$t_K\big[Q^{(\pm)}_{\lambda\nu}\big] \equiv \langle 0|[Q^{(\pm)}_{\lambda\nu}, X^{(\pm)\dagger}_K]|0\rangle = \langle {\rm f}|Q^{(\pm)}_{\lambda\nu}|{\rm i}\rangle_{\rm RPA}, \quad (K, \nu \geq 0). \tag{3.12}$$

Note that only the signature component $r = (-1)^\lambda$ ($(-1)^{\lambda'}$) exists for $\nu = 0$ ($K = 0$), and the RPA transition matrix elements vanishes if the signature mismatches between the state and the operator. By using the fact that

$$t_K\big[Q^{(+)}_{\lambda\nu}\big] = -\,t_K\big[Q^{(-)}_{\lambda\nu}\big] \times (1 + O(\omega^2_{\rm rot})), \quad (K, \nu > 0), \tag{3.13}$$

due to the phase convention (2.7) and because of the degeneracy of $r = \pm 1$ RPA modes at $\omega_{\rm rot} = 0$, the intrinsic matrix elements in Eq.(2.14) are given by

$$({\rm f}|\widehat{m}^{(0)}_{\lambda,-K}|{\rm i}) = \Big[\, t_K\big[Q^{(\sigma)}_{\lambda K}\big]\Big]_0, \tag{3.14a}$$

$$({\rm f}|\widehat{m}^{(\pm 1)}_{\lambda,-K\pm 1}|{\rm i}) = \sqrt{\frac{2-\delta_{K0}}{2(1+\delta_{K,\pm 1})}}\,\frac{1}{\mathcal{J}}\Big[\frac{d\,t_K\big[Q^{(\sigma)}_{\lambda|-K\pm 1|}\big]}{\omega_{\rm rot}}\Big]_0, \tag{3.14b}$$

where

$$\sigma = \begin{cases} + & \text{for} \quad \lambda = \text{even}, \\ - & \text{for} \quad \lambda = \text{odd}. \end{cases} \tag{3.14c}$$

The GIR is then given by Eq.(3.8) with $K_{\rm i} = K$, $K_{\rm f} = 0$ and $\Delta K = -K \leq 0$, namely,

$$\begin{aligned} \mathcal{T}(\lambda : K\,{\rm i} \to 0\,{\rm f}) =& \langle I_{\rm i} K \lambda\,(-K)|I_{\rm f} 0\rangle\, Q_t \left(1 + q\,[I_{\rm f}(I_{\rm f}+1) - I_{\rm i}(I_{\rm i}+1)]\,\right) \\ &+ \sqrt{I_{\rm f}(I_{\rm f}+1)}\,\langle I_{\rm i} K \lambda\,(-K-1)|I_{\rm f}\,(-1)\rangle\, Q'_t, \end{aligned} \tag{3.15}$$

and using $C_{\rm if} = \sqrt{2-\delta_{K0}}$ for $K_{\rm i} = K$ and $K_{\rm f} = 0$,

$$\begin{aligned} Q_t =& \sqrt{2-\delta_{K0}}\,\Big[\, t_K\big[Q^{(\sigma)}_{\lambda K}\big]\Big]_0 - \frac{2-\delta_{K0}}{\sqrt{2}}\,\frac{\sqrt{(\lambda-K)(\lambda+K+1)}}{2\mathcal{J}}\Big[\frac{d\,t_K\big[Q^{(\sigma)}_{\lambda,K+1}\big]}{d\omega_{\rm rot}}\Big]_0 \\ &+ \frac{2-\delta_{K0}}{\sqrt{2(1+\delta_{K1})}}\,\frac{2K^2 + (\lambda-K)(\lambda+K+1)}{\sqrt{(\lambda+K)(\lambda-K+1)}}\,\frac{1}{2\mathcal{J}}\Big[\frac{d\,t_K\big[Q^{(\sigma)}_{\lambda|K-1|}\big]}{d\omega_{\rm rot}}\Big]_0, \\ Q'_t =& \frac{2-\delta_{K0}}{\sqrt{2}}\,\frac{1}{\mathcal{J}}\Big[\frac{d\,t_K\big[Q^{(\sigma)}_{\lambda,K+1}\big]}{d\omega_{\rm rot}}\Big]_0 \end{aligned}$$

$$- \frac{2-\delta_{K0}}{\sqrt{2(1+\delta_{K1})}} \sqrt{\frac{(\lambda-K)(\lambda+K+1)}{(\lambda+K)(\lambda-K+1)}} \frac{1}{\mathcal{J}} \Big[\frac{d\, t_K \big[Q^{(\sigma)}_{\lambda|K-1|} \big]}{d\omega_{\rm rot}} \Big]_0 ,$$

$$q = \frac{2-\delta_{K0}}{\sqrt{2(1+\delta_{K1})}} \frac{1}{\sqrt{(\lambda+K)(\lambda-K+1)}} \frac{1}{\mathcal{J}} \Big[\frac{d\, t_K \big[Q^{(\sigma)}_{\lambda|K-1|} \big]}{d\omega_{\rm rot}} \Big]_0 / Q_t . \tag{3.16}$$

It is easy to see that $Q'_t = 0$ for both $K = 0$ and $K = \lambda$ cases. These formula were employed in Ref. 13) to analyze the $E1$ and $E3$ transitions from the octupole vibrational bands.

3-5. $E2$ and $M1$ Transitions from quadrupole-vibrational bands

In the previous subsection, we have given the GIR, Eqs.(3.15) and (3.16), for arbitrary type of transitions to the ground band in even-even nuclei ($K_{\rm f} = 0$), assuming the simple RPA method. For odd nuclei ($K_{\rm f} \neq 0$), the most important corrections to the RPA may be taken into account by means of particle-vibration coupling in the rotating frame.[14,15] In principle, the approach presented in §2 is general and applicable to such cases.

Since the occurrence of collective-quadrupole transitions is a dominant feature of low-lying nuclear spectra, we now consider the interband $E2$ and $M1$ transitions from the quadrupole-vibrational bands. In deformed nuclei, we have the β and γ vibrations which transfer the K-quantum number by 0 and ± 2 units, respectively. In this subsection, we assume that the band on which the quadrupole vibration is built has non-zero K-quantum number, and we do not assume the simple RPA method.

Let us start with the β vibration; the β vibration neither transfers the K-quantum number nor changes the signature-quantum number. The GIR for $E2$ transitions is given by the general formula in Eq.(3.8) with $\lambda = 2$, $K_{\rm i} = K_{\rm f} = K > 0$, but since $\Delta K = 0$ we must choose either the upper or lower expression; we here take the upper one. Then the intrinsic parameters (3.9) are explicitly given in terms of the matrix elements in the signature-good states as

$$Q_t = \Big[\langle {\rm f}|Q^{(+)}_{20}|{\rm i}\rangle \Big]_0 + \frac{\sqrt{3}}{\mathcal{J}} \Big[\frac{d\langle \bar{\rm f}|Q^{(-)}_{21}|{\rm i}\rangle}{d\omega_{\rm rot}} \Big]_0 , \quad Q'_t = -\frac{\sqrt{2}}{\mathcal{J}} \Big[\frac{d\langle \bar{\rm f}|Q^{(-)}_{21}|{\rm i}\rangle}{d\omega_{\rm rot}} \Big]_0 ,$$

$$q = \frac{1}{2\sqrt{3}\mathcal{J}} \Big[\frac{d\langle {\rm f}|Q^{(+)}_{21}|{\rm i}\rangle}{d\omega_{\rm rot}} + \frac{d\langle \bar{\rm f}|Q^{(-)}_{21}|{\rm i}\rangle}{d\omega_{\rm rot}} \Big]_0 / Q_t , \quad (\Delta K = 0). \tag{3.17}$$

Here we have used the identities between the matrix elements shown in Appendix B. In the simple RPA method, where $|\mathrm{i}\rangle = X^\dagger_\beta|\mathrm{f}\rangle$ and $X^\dagger_\beta$ is the creation operator of the β vibration, the matrix elements of the signature $(-)$ operator, i.e. $Q^{(-)}_{21}$ term in Eq.(3.17), vanishes.

Although the interband $M1$ transitions are not necessarily enhanced by the quadrupole collectivity, the GIR for them can be obtained in the same way. The intrinsic parameters in this case are,

$$\begin{aligned} Q_t &= \Big[\langle\bar{\mathrm{f}}|\mu_z|\mathrm{i}\rangle\Big]_0 - \frac{1}{\mathcal{J}}\Big[\frac{d\langle\mathrm{f}|\mu_x|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0, \quad Q'_t = \frac{\sqrt{2}}{\mathcal{J}}\Big[\frac{d\langle\mathrm{f}|\mu_x|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0, \\ q &= -\frac{1}{2\mathcal{J}}\Big[\frac{d\langle\bar{\mathrm{f}}|i\mu_y|\mathrm{i}\rangle}{d\omega_{\rm rot}} + \frac{d\langle\mathrm{f}|\mu_x|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0/Q_t, \quad (\Delta K = 0). \end{aligned} \tag{3.18}$$

Again, μ_z and $i\mu_y$ terms in Eq.(3.18) vanish if we use the simple RPA method. Note that the relative phase between Eqs.(3.17) and (3.18) is meaningful so that the sign of $M1/E2$ mixing ratio is also determined.

Now let us consider the case of the γ vibration; the γ vibration transfers the K-quantum number by $\Delta K = \pm 2$, and two independent modes exist, whose signatures are $\alpha = 0$ and 1. Therefore, if $K_{\rm f} \neq 0$, there are two (strongly-coupled) bands with $K_{\rm i} = K_{\rm f} + 2$ and $K_{\rm f} - 2$ for a given final state on which the γ-vibrational modes are built. Assuming that both initial and final states have positive K-quantum number at $\omega_{\rm rot} = 0$ i.e. $K_{\rm f} > 2$ for simplicity, we obtain the intrinsic parameters for $E2$ transitions between the γ band and the ground-state band;

$$\begin{aligned} Q_t &= \sqrt{2}\Big[\langle\mathrm{f}|Q^{(+)}_{22}|\mathrm{i}\rangle\Big]_0 \pm \frac{\sqrt{2}\,(K_{\rm i}+K_{\rm f})}{\mathcal{J}}\Big[\frac{d\langle\mathrm{f}|Q^{(+)}_{21}|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0, \quad Q'_t = 0, \\ q &= \frac{1}{\sqrt{2}\mathcal{J}}\Big[\frac{d\langle\mathrm{f}|Q^{(+)}_{21}|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0/Q_t, \quad (\Delta K = \mp 2), \end{aligned} \tag{3.19}$$

which enter in the GIR (3.8), or equivalently in (3.10) with $n = 0$. Here it is understood that $|\mathrm{f}\rangle$ and $|\mathrm{i}\rangle$ have the same signature and the identities in Appendix B are used.

The interband $M1$ transitions from the γ band is K-forbidden by order $n = 1$, and the GIR can be obtained in the same way as for the $E2$ transitions. Again, assuming both initial and final states have non-zero K-quantum number at $\omega_{\rm rot} = 0$, the GIR is given by Eq.(3.10) with the intrinsic parameters

$$Q_t = -\frac{\sqrt{2}}{\mathcal{J}}\Big[\frac{d\langle\bar{\mathrm{f}}|i\mu_y|\mathrm{i}\rangle}{d\omega_{\rm rot}}\Big]_0 \pm \frac{K_{\rm i}+K_{\rm f}}{2\sqrt{2}\mathcal{J}^2}\Big[\frac{d^2\langle\bar{\mathrm{f}}|\mu_z|\mathrm{i}\rangle}{d\omega^2_{\rm rot}}\Big]_0,$$

$$q = \frac{1}{2\sqrt{2}\mathcal{J}^2}\Big[\frac{d^2\langle \bar{\mathrm{f}}|\mu_z|\mathrm{i}\rangle}{d\omega_{\rm rot}^2}\Big]_0 / Q_t, \quad (\Delta K = \mp 2). \tag{3.20}$$

Again, the relative sign between Eqs.(3.19) and (3.20) is related to the sign of the $M1/E2$ mixing ratio and is determined by the phase convention (2.7).

Before ending this section, let us discuss the $M1$ and $E2$ transitions from β and γ bands to the ground-state band of even-even nuclei ($K_{\rm f} = 0$). A large body of experimental data exists and the amplitudes are relatively easy to calculate by means of the simple RPA method. Although the general results have been already given in §3-4 for the K-allowed vibrational transitions, here we summarize the formulae for the specific case under consideration. Using the RPA transition amplitudes, $t_\beta\big[*\big]$ and $t_\gamma\big[*\big]$, for the β and γ vibrations, respectively, it is straightforward to show that the formula can be unified into

$$\begin{aligned}\mathcal{T}(E2 : K\,\mathrm{i} \to 0\mathrm{f}) =& \langle I_{\rm i} K 2\,(-K)|I_{\rm f} 0\rangle\, Q_t \\ & \times \big(1 + q\,[I_{\rm f}(I_{\rm f}+1) - I_{\rm i}(I_{\rm i}+1)]\,\big), \end{aligned} \tag{3.21}$$

$$\begin{aligned}\mathcal{T}(M1 : K\,\mathrm{i} \to 0\mathrm{f}) =& \sqrt{I_{\rm f}(I_{\rm f}+1)}\,\langle I_{\rm i} K 1\,(1-K)|I_{\rm f} 1\rangle\, M_t \\ & \times \big(1 + m\,[I_{\rm f}(I_{\rm f}+1) - I_{\rm i}(I_{\rm i}+1)]\,\big), \end{aligned} \tag{3.22}$$

with

$$Q_t^{(\beta)} = \Big[\, t_\beta\big[Q_{20}^{(+)}\big]\Big]_0, \quad q^{(\beta)} \equiv \frac{1}{2\sqrt{3}\mathcal{J}}\Big[\frac{d\, t_\beta\big[Q_{21}^{(+)}\big]}{d\omega_{\rm rot}}\Big]_0 / Q_t^{(\beta)}, \tag{3.23a}$$

$$M_t^{(\beta)} = \frac{\sqrt{2}}{\mathcal{J}}\Big[\frac{d\, t_\beta\big[\mu_x\big]}{d\omega_{\rm rot}}\Big]_0, \quad m^{(\beta)} = 0, \tag{3.23b}$$

$$Q_t^{(\gamma)} = \sqrt{2}\Big[\, t_\gamma\big[Q_{22}^{(-)}\big]\Big]_0 + \frac{2\sqrt{2}}{\mathcal{J}}\Big[\frac{d\, t_\gamma\big[Q_{21}^{(-)}\big]}{d\omega_{\rm rot}}\Big]_0, \quad q^{(\gamma)} = \frac{1}{\sqrt{2}\mathcal{J}}\Big[\frac{d\, t_\gamma\big[Q_{21}^{(-)}\big]}{d\omega_{\rm rot}}\Big]_0 / Q_t^{(\gamma)}, \tag{3.24a}$$

$$M_t^{(\gamma)} = \frac{\sqrt{2}}{\mathcal{J}}\Big[\frac{d\, t_\gamma\big[i\mu_y\big]}{d\omega_{\rm rot}}\Big]_0 - \frac{1}{\sqrt{2}\mathcal{J}^2}\Big[\frac{d^2\, t_\gamma\big[\mu_z\big]}{d\omega_{\rm rot}^2}\Big]_0, \quad m^{(\gamma)} = -\frac{1}{2\sqrt{2}\mathcal{J}^2}\Big[\frac{d^2\, t_\gamma\big[\mu_z\big]}{d\omega_{\rm rot}^2}\Big]_0 / M_t^{(\gamma)}, \tag{3.24b}$$

where $K_{\rm i} \equiv K = 0$ (2) for the β (γ) vibrations, respectively, and we have used Eq.(3.13) so as to make the sign of the $M1/E2$ mixing ratio apparent (although the overall sign is arbitrary chosen). These formulae have also been applied to experimental data in Ref.13). It should be mentioned that the GIR (3.21) for the γ-vibrational band was derived in

Ref.5), but because the operator ordering was not considered the second term in $Q_t^{(\gamma)}$ did not appear.

§4 Examples of Numerical Calculations

In order to show the usefulness of our new approach to the intensity relationship presented in the previous sections, we have performed numerical calculations for transitions from quadrupole- and octupole-vibrational bands to the ground-state band. Since the results for ^{238}U were already reported in Ref.13), we have selected even-even nuclei in the rare-earth region, ^{156,158,160}Gd, ^{160,162,164}Dy, and ^{166,168,170}Er, for which comprehensive data have been reported[16,17] for the transition probabilities from both quadrupole- and octupole-vibrational bands.

The RPA calculations for the collective vibrational bands were performed with the cranked-Nilsson mean field (see the previous works[5,9,11,12] for details).* The standard Nilsson potential with single-stretched $\boldsymbol{ll}$ and $\boldsymbol{ls}$ terms[18] was adopted, and we used the experimental quadrupole deformations[19] which were deduced from the intrinsic Q-moments in the ground bands. The monopole pairing, quadrupole, and octupole separable forces were employed as residual interactions. The force strengths of these interactions was determined at zero rotational frequency: The experimental even-odd mass difference, which was calculated by the third-order-difference formula applied to the 1993 Mass Table,[20] was used to fix the monopole-pairing strength. The quadrupole and octupole force strengths for each K value were chosen so as to reproduce the observed band-head energies of the corresponding collective vibrational bands. The requirement of a zero-energy solution (Nambu-Goldstone mode) was used for the $K = 1$ component of the quadrupole interaction.

In order to estimate the lowest-order corrections of the Coriolis coupling, we calculated the RPA transition amplitudes at finite (infinitesimal) frequency. The pairing correlations were selfconsistently calculated at finite frequency, while the deformation parameters were assumed to be constant. In principle, the moments of inertia, which are necessary for

* The modified treatment of the cranking term,[12] which eliminates the spurious velocity dependence of the mean field, is not used in this paper.

calculating the lowest-order corrections, could be obtained in the cranking model. However, it is well-known that the Nilsson model without higher-multipole pairing correlations generally underestimates moments of inertia near the ground state. Thus, we use the experimental values estimated from excitation energies of the first 2^+ states in the ground band. The parameters used in the calculation are summarized in Table 1 (the band-head energies of octupole bands are listed in Table 3).

The transition amplitudes are often overestimated in RPA calculations with the full model space. We have used three major shells as the active model space for both neutrons and protons, i.e. $N_{\rm osc} = 4 - 6$ for neutrons and $N_{\rm osc} = 3 - 5$ for protons, which gives good overall agreement[5] for absolute values of transition amplitudes. Although these absolute values depend on the size of model space, the quantities representing effects of the Coriolis coupling (q and m in Eqs.(3.15), (3.21), and (3.22)) are not sensitive to it.

Table 2 shows the calculated results for the GIR parameters in Eqs.(3.21–24), associated with the transitions from the β ($K = 0$) and γ ($K = 2$) vibrational bands, where they are compared with experimental data for $I^\pi_{\rm i} = 2^+$ states. The agreement of the Q_t parameters for the γ vibrations is excellent, while the experimental values for the β vibrations are smaller by about factor two (except in ^{156}Gd). The reduction and the fragmentation of measured collectivity of β vibrations compared with RPA calculations is well-known in this region,[16,17] and calls for deeper understanding of the low-lying quadrupole-vibrational modes. The lowest-order corrections of the Coriolis coupling are generally small for $E2$ amplitudes: The calculated values of the parameter q are always positive and typically $0.02 - 0.03$, which qualitatively agrees with the experimental observations. Again the agreement is better for the γ band than for the β band.

The intrinsic parameters Q_t and q for the γ-vibrational band in some nuclei were calculated previously in Ref.5). The differences in those results compared with the present calculations are due to the fact that the operator ordering was not considered previously. As is clear from Eq.(3.24), the extra contribution coming from the "contraction" due to the ordering (2.12) can be written as $4\,Q_t^{(\gamma)} q^{(\gamma)}$, which increases the leading-order amplitude $Q_t^{(\gamma)}$ by about 5-10% and consequently reduces $q^{(\gamma)}$ in comparison with the old results. In Ref.5), it was also demonstrated that the ambiguity of the moment of inertia results in an

uncertainty of about 10% in the q parameters.

The $M1$ transitions from quadrupole-vibrational bands to the ground-state bands are expected to be small because the coherence property of the quadrupole phonon is different from that of the $M1$ operator. This comes about because the $M1$ operator has the opposite sign to the $E2$ operator with respect to H-conjugation.[2] That the $M1$ transitions is weak is apparent in both the calculations and the experiment. In particular the $E2/M1$ mixing ratios are large, being typically $|\delta| > 10$ for the γ band.[16,17] Although the $M1$ amplitudes are small, the agreement between theory and experiment for the absolute values of the main amplitude parameters M_t is good. It should be noted that, experimentally, the sign of the mixing amplitude is negative for all the γ bands in the table, while the calculation indicates that the sign changes from negative to positive as one moves up to heavier isotopes. It is worthwhile mentioning that the $M1$ transitions from the quadrupole-vibrational bands to the yrast bands are predicted to become large at high spin.[5] This is because the strong Coriolis coupling changes the nature of the quadrupole vibration into a wobbling-like motion and the coherence property of the phonon begins to favor the $M1$ operator. The properties of $M1$ transitions from high-spin vibrational excitations is an interesting problem for the future.

In Table 3, we show the calculated results for the octupole-vibrational bands (see Eq.(3.16) with $\lambda = 3$). Since there are not enough data to extract the GIR parameters, we have compared the calculated and experimental amplitudes of the stretched vibrational transitions, $I^\pi = 3^-_K \to 0^+_{\rm g}$. It has been discussed in Ref. 21) that Coriolis mixing amongst the octupole vibrations with different K is important even at low spins. The measured relative intensities of transitions associated with the different K modes cannot be understood without Coriolis-coupling effects. The reason why Coriolis coupling for the $E3$ transitions ($3^-_K \to 0^+_{\rm g}$) is more important than for the $E2$ ($2^+_{\beta,\gamma} \to 0^+_{\rm g}$) can be understood as follows: In principle, all members of the octupole multiplet with $K = 0$, 1, 2, and 3 can occur as low-energy modes of excitation, but there is no low-lying quadrupole vibration with $K = 1$. Therefore, the octupole bands are expected to be mixed with each other more strongly than are the quadrupole bands ($\Delta K = 2$). In addition, for the stretched vibrational transitions to the ground state, the factor multiplying the Coriolis-coupling

parameter q in Eq.(3.15), namely $-\lambda(\lambda+1)$, takes the value -12 for the octupole bands and -6 for the quadrupole bands. Typically, the lowest-order coupling changes the $E2$ amplitudes by less than $\sim$10%, but it can change the $E3$ amplitudes by more than 50%.

Because we are restricting this discussion to the $I_{\rm f}=0$ case, the second (Q'_t) term in Eq.(3.15) does not contribute to the $E3$ amplitudes. However, the values of parameter Q'_t in Table 3 provide qualitative information concerning the magnitude of the Coriolis mixing. Since our treatment is perturbative in its nature, the results will not be reliable if the Coriolis coupling is too strong. A naive criterion for the validity of our approach with respect to the Coriolis coupling may be given by $|Q'_t| \ll |Q_t|$. This criterion is broken for some nuclei in which two octupole modes with adjacent K-quantum numbers are almost degenerate, e.g. $K=1$ and 2 in ^{160}Dy, $K=0$ and 1 in ^{164}Dy, and $K=2$ and 3 in ^{168}Er (there is a similar degeneracy for the β and γ bands in ^{158}Gd). In those cases, the lowest-order correction $({\rm f}|\widehat{m}^{(\pm1)}_{3\Delta K\pm1}|{\rm i})$ is comparable to the main amplitude $({\rm f}|\widehat{m}^{(0)}_{3\Delta K}|{\rm i})$, therefore the second and third terms in the expression of Q_t (3.16) will be even larger than the first (leading) term, indicating a breakdown of perturbation theory.

Table 3 also presents amplitudes calculated without the effect of the Coriolis coupling (the column "cal.(0)"). As is clear from the table, the calculated values of the parameter q for the lowest octupole states are always negative (except for ^{160}Dy). This has the effect of concentrating $E3$ strength onto the lowest state. This property agrees with the results of Ref.21). Except for the "singular" cases mentioned above, agreement with the experimental data is significantly improved by the lowest-order corrections.

§5. Concluding Remarks

In this paper we have developed a new general method to calculate the intrinsic parameters of the generalized intensity relations for transition probabilities. The intrinsic matrix elements in the original formulation of the unified model[2] are related to matrix elements in the microscopic cranking calculations. Useful formula for the electromagnetic transition amplitudes are presented.

In our approach, transition probabilities are microscopically calculated by the cranking model. Since the geometry of angular momentum (Clebsch-Gordan coefficients) is fully

taken into account, our results are applicable to low-spin states. We have presented the numerical results of cranked RPA calculations giving transition amplitudes between vibrational and ground-state bands. These examples clearly show that our formalism gives an improved understanding of the transition rates between these low-lying collective motions.

The generalized intensity relation is derived by a perturbative treatment of the intrinsic-angular-momentum operator (I expansion).[2] In principle, the correction terms to the leading-order relation could be evaluated by explicitly calculating the perturbation of the Coriolis couplings in the unified model. This may be relatively easy for single-particle states, but it becomes much more difficult for correlated states such as collective vibrations. In contrast, our method gives a systematic and practical means to account for the Coriolis coupling in terms of the cranking formalism.

Until now, the cranking model has been considered unreliable in calculating transition matrix elements, especially at low spin. This is indeed so if the angular momentum is treated only on average, as in the semiclassical approximation. In the high-spin limit, where the rotational motion is uniform around one of the principal axis of the mean field, the effect of angular-momentum algebra is not important because the Clebsch-Gordan coefficients can be replaced by their asymptotic values ($1/I$ expansion).[4] The formalism developed in this paper gives a complementary method which is a good approximation for low spins, even though it uses the same apparatus, namely the cranking model.

Although the idea of the present approach is general, we have restricted its application to the lowest-order corrections because the problem of operator ordering appears to be more complex for the higher orders. In order to extend the formulation to the higher orders, a more systematic quantum-mechanical treatment of the microscopic cranking model may be necessary.[3] This is an important future problem.

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Table 1. Basic parameters used in the calculations. The neutron and proton pairing gaps ($\Delta_{\rm n,p}$) are obtained by third differences from the 1993 Mass Table.[20] The Nilsson-deformation parameters (ϵ_2) are taken from Ref.19). The moments of inertia ($\mathcal{J}_{\rm g}$) are obtained from the first 2^+ energies of the ground-state bands. The band-head energies of the γ and β vibrations ($E_{\gamma,\beta}$) are taken from the compilation of Ref.22).

Nucl.	$\Delta_{\rm n}$ [MeV]	$\Delta_{\rm p}$ [MeV]	ϵ_2	$\mathcal{J}_{\rm g}$ [$\hbar^2$/MeV]	E_γ [keV]	E_β [keV]
^{156}Gd	1.070	0.960	0.274	33.7	1154	1050
^{158}Gd	0.892	0.878	0.282	37.7	1187	1196
^{160}Gd	0.831	0.871	0.287	39.9	988	1378
^{160}Dy	0.967	0.978	0.271	34.6	966	1275
^{162}Dy	0.917	0.930	0.270	37.2	888	1131
^{164}Dy	0.832	0.875	0.275	40.9	762	1655
^{166}Er	0.966	0.877	0.272	37.2	786	1460
^{168}Er	0.776	0.857	0.271	37.6	821	1217
^{170}Er	0.708	0.797	0.268	38.2	934	891

Table 2. Comparison of the GIR parameters for $E2$ and $M1$ transitions [Eqs.(3.23) and (3.24)] from the $\gamma\,(K=2)$ and $\beta\,(K=0)$ bands to the ground-state band. Experimental values[16,17] for Q_t (M_t) are extracted from $B(E2(M1);2^+_{\beta,\gamma}\to 2^+_{\rm g})$ and those for q are extracted both from $B(E2;2^+_{\beta,\gamma}\to 0^+_{\rm g})$ (indicated by "exp.(1)") and $B(E2;2^+_{\beta,\gamma}\to 4^+_{\rm g})$ ("exp.(2)"). Note that the relative phase between $E2$ and $M1$ matrix elements (the $E2/M1$ mixing ratio) is meaningful in both the experiment and the calculation.

Nucl.		Q_t [eb]		q			M_t [$\mu_{\rm N}$]		m
		cal.	exp.	cal.	exp.(1)	exp.(2)	cal.	exp.	cal.
^{156}Gd	γ	0.391	0.352	0.024	0.009	0.025	−0.0059	−0.0059	0.078
	β	0.171	0.267	0.042	0.088	−0.023	−0.0115	−0.0066	
^{158}Gd	γ	0.329	0.333	0.020	0.021	−0.012	−0.0026	−0.0128	0.120
	β	0.132	0.066	0.015	−0.061	0.057	0.0448	−0.0191	
^{160}Gd	γ	0.328	0.323	0.021	0.014	0.0026	0.0008	−0.0009*	−0.375
	β	0.116		0.012			0.0515		
^{160}Dy	γ	0.412	0.395	0.025	0.019	0.012	−0.0037	−0.0035	0.128
	β	0.160	0.114	0.022	−0.033	0.011	0.0241	−0.0245	
^{162}Dy	γ	0.397	0.395	0.024	0.022	0.032	0.0002	−0.0011*	−2.890
	β	0.226		0.021			0.0333		
^{164}Dy	γ	0.412	0.379	0.021	0.019	0.035	0.0070	−0.0018	−0.050
	$\beta^\dagger$	0.010		0.624			−0.1588		
^{166}Er	γ	0.442	0.436	0.022	0.024	0.030	0.0019	−0.0039	−0.227
	β	0.193		0.031			−0.0370		
^{168}Er	γ	0.439	0.419	0.016	0.020	0.007	0.0094	−0.0039*	−0.031
	β	0.158		0.037			−0.0367		
^{170}Er	γ	0.421	0.363	0.013	0.020	0.004	0.0119	−0.0014*	−0.018
	β	0.208	0.099	0.023	0.016	0.035	0.0147	0.0129	

*) Only a lower limit for the absolute value is given.

†) An almost pure two quasiparticle state in the calculation using the energy in Table 1.

Table 3. Calculated GIR parameters (Q_t, Q'_t, q) and $E3$ amplitudes (cal.(0), cal.) in Eq.(3.15) for $3^- \to 0^+_{\rm g}$ transitions from the octupole-vibrational bands. "cal.(0)" indicates the calculated $E3$ amplitudes neglecting the Coriolis coupling. Observed band-head energies[22] (E_K) and $E3$ amplitudes[16,17] (exp.) are also listed.

Nucl.	K	E_K	$Q_t\,[\mathrm{eb}^{\frac{3}{2}}]$	$Q'_t\,[\mathrm{eb}^{\frac{3}{2}}]$	q	$\mathcal{T}(E3:3^- \to 0^+_{\rm g})\,[\mathrm{eb}^{\frac{3}{2}}]$		
			cal.	cal.	cal.	cal.(0)	cal.	exp.
^{156}Gd	0	1367	0.161	0	0.116	0.061	0.024	0.043*
	1	1243	0.111	0.065	−0.227	0.090	0.156	0.156
	2	1780	0.284	−0.057	0.022	0.071	0.079	
	3	1934	0.342	0	0.042	0.081	0.065	
^{158}Gd	0	1264	0.150	0	0.045	0.057	0.026	0.018
	1	977	0.210	0.020	−0.040	0.094	0.117	0.133
	2	1794	0.199	−0.031	0.004	0.060	0.072	
	3		0.130	0	0.050	0.027	0.019	
^{160}Gd	0	1225	0.146	0	−0.019	0.055	0.067	0.130
	1	1569	0.139	0.008	0.027	0.056	0.036	
	2	1016	0.281	−0.008	−0.011	0.107	0.120	
	3	1463	0.314	0	0.023	0.094	0.086	
^{160}Dy	0		0.063	0	0.063	0.024	0.006	
	1	1286	0.533	−0.474	0.005	0.083	0.190	0.096†
	2	1265	0.619	−0.302	0.204	0.097	0.340	0.156
	3	2075	0.224	0	0.018	0.071	0.066	
^{162}Dy	0	1276	0.157	0	−0.023	0.059	0.076	0.070‡
	1	1637	0.175	0.002	0.034	0.065	0.039	
	2	1148	0.260	−0.009	−0.016	0.101	0.118	0.122
	3	1571	0.324	0	0.026	0.094	0.085	
^{164}Dy	0	1675	0.120	0	0.294	0.045	0.115	
	1	1637	0.072	−0.147	0.587	0.077	0.165	
	2	977	0.277	−0.0001	−0.011	0.109	0.118	0.112
	3	1766	0.210	0	0.021	0.065	0.060	

Table 3. (*continued*).

Nucl.	K	E_K	$Q_t\,[\mathrm{eb}^{\frac{3}{2}}]$ cal.	$Q'_t\,[\mathrm{eb}^{\frac{3}{2}}]$ cal.	q cal.	$\mathcal{T}(E3:3^- \to 0^+_{\mathrm{g}})\,[\mathrm{eb}^{\frac{3}{2}}]$ cal.(0)	cal.	exp.
^{166}Er	0	1663	0.143	0	−0.065	0.054	0.096	0.068
	1	1831	0.251	−0.024	0.051	0.076	0.036	
	2	1458	0.228	−0.006	−0.022	0.091	0.109	0.093
	3	1918	0.196	0	0.033	0.053	0.045	
^{168}Er	0	1786	0.104	0	0.031	0.039	0.025	
	1	1359	0.191	−0.008	−0.020	0.069	0.089	0.078
	2	1569	0.031	−0.211	−0.106	0.081	0.027	0.085
	3	1542	0.642	0	0.147	0.079	0.186	
^{170}Er	0	1539	0.051	0	0.008	0.019	0.017	
	1	1266	0.074	0.002	−0.028	0.030	0.037	
	2		0.124	0.023	−0.005	0.059	0.049	
	3	1304	0.221	0	−0.017	0.096	0.100	

*) Only the upper limit is given.
†) Data from Ref.23).
‡) Data from Ref.24).

Appendix A

In this appendix we discuss the possible ordering of operators, $I_\pm$ and $\mathcal{D}^\lambda_{\mu\nu}$, in the "quantization" procedure, and show that the symmetrized ordering adopted in Eq.(2.12b) is the most appropriate.

We consider here the simple case where the λ-pole transitions occur from the λ-pole vibrational states ($K_{\rm i} = K$, $I_{\rm i} = \lambda$) to the ground state ($K_{\rm f} = I_{\rm f} = 0$). The Coriolis coupling is assumed to be effective only within the λ-pole vibrational multiplets with different K-quantum numbers ($K = -\lambda, -\lambda+1, \cdots, \lambda$). We impose the following two conditions; (i) detailed balance is satisfied, i.e.,

$$|\langle 00||\mathcal{M}(\lambda)||K\lambda\rangle| = |\langle K\lambda||\mathcal{M}(\lambda)||00\rangle|, \tag{A.1}$$

and (ii) the sum of the transition strengths for the modes with different K values is preserved within the first order in $\omega_{\rm rot}$, i.e.,

$$\sum_{K=-\lambda}^{\lambda} |\langle 00||\mathcal{M}(\lambda)||K\lambda\rangle_{\rm unsym}|^2 = \Big[\sum_{K=-\lambda}^{\lambda} |\langle 00||\mathcal{M}(\lambda)||K\lambda\rangle_{\rm unsym}|^2\Big]_0 + O(\omega_{\rm rot}^2), \tag{A.2}$$

where $\langle||*||\rangle_{\rm unsym}$ means that the K-good representation for the wave functions is used, i.e., the symmetrization according to $\mathcal{R}$ conjugation is not considered. After the summation with respect to K in Eq.(A.2), this treatment gives the same result as the case where the $\mathcal{R}$ conjugation is taken into account. The symbol $[*]_0$ means that it is obtained neglecting the Coriolis coupling ($\omega_{\rm rot} = 0$).

From the first requirement (A.1), it is easy to see that only the following orderings are allowed:

$$\mathcal{M}(\lambda\mu)_{\Delta K}$$

$$= \begin{cases} \widehat{m}^{(0)}_{\lambda\Delta K}\,\mathcal{D}^\lambda_{\mu\Delta K} + \widehat{m}^{(+1)}_{\lambda\Delta K+1}\dfrac{1}{2}\big\{I_+, \mathcal{D}^\lambda_{\mu,\Delta K+1}\big\} + \widehat{m}^{(-1)}_{\lambda\Delta K-1}\dfrac{1}{2}\big\{I_-, \mathcal{D}^\lambda_{\mu,\Delta K-1}\big\}, & \text{(A.3a)} \\ \widehat{m}^{(0)}_{\lambda\Delta K}\,\mathcal{D}^\lambda_{\mu\Delta K} + \widehat{m}^{(+1)}_{\lambda\Delta K+1}\,I_+\mathcal{D}^\lambda_{\mu,\Delta K+1} + \widehat{m}^{(-1)}_{\lambda\Delta K-1}\,\mathcal{D}^\lambda_{\mu,\Delta K-1}I_-, & \text{(A.3b)} \\ \widehat{m}^{(0)}_{\lambda\Delta K}\,\mathcal{D}^\lambda_{\mu\Delta K} + \widehat{m}^{(+1)}_{\lambda\Delta K+1}\,\mathcal{D}^\lambda_{\mu,\Delta K+1}I_+ + \widehat{m}^{(-1)}_{\lambda\Delta K-1}\,I_-\mathcal{D}^\lambda_{\mu,\Delta K-1}. & \text{(A.3c)} \end{cases}$$

We have adopted the ordering of Eq.(A.3a) in the "quantization" (2.12b). The ordering given in Eq.(A.3b) or (A.3c) leads a different result for the GIR.

Next, in order to check the second requirement (A.2), we calculate the effect of the Coriolis coupling in the lowest order of $\omega_{\rm rot}$. Using Eq.(2.14), we obtain

$$\sum_{K=-\lambda}^{\lambda}|\langle 00||\mathcal{M}(\lambda)||K\lambda\rangle_{\rm unsym}|^2-\Big[\sum_{K=-\lambda}^{\lambda}|\langle 00||\mathcal{M}(\lambda)||K\lambda\rangle_{\rm unsym}|^2\Big]_0$$

$$\approx\begin{cases}-\dfrac{1}{\mathcal{J}}\displaystyle\sum_{K=-\lambda}^{\lambda}\sqrt{(\lambda-K)(\lambda+K+1)}\\ \qquad\times\Big[\dfrac{d\,t_K\big[Q_{\lambda,-K-1}\big]}{d\omega_{\rm rot}}\,t_K\big[Q_{\lambda,-K}\big]+\dfrac{d\,t_{K+1}\big[Q_{\lambda,-K}\big]}{d\omega_{\rm rot}}\,t_{K+1}\big[Q_{\lambda,-K-1}\big]\Big]_0, & \text{(A.4a)}\\ -\dfrac{2}{\mathcal{J}}\displaystyle\sum_{K=-\lambda}^{\lambda}\sqrt{(\lambda-K)(\lambda+K+1)}\,\Big[\dfrac{d\,t_K\big[Q_{\lambda,-K-1}\big]}{d\omega_{\rm rot}}\,t_K\big[Q_{\lambda,-K}\big]\Big]_0, & \text{(A.4b)}\\ -\dfrac{2}{\mathcal{J}}\displaystyle\sum_{K=-\lambda}^{\lambda}\sqrt{(\lambda-K)(\lambda+K+1)}\,\Big[\dfrac{d\,t_{K+1}\big[Q_{\lambda,-K}\big]}{d\omega_{\rm rot}}\,t_{K+1}\big[Q_{\lambda,-K-1}\big]\Big]_0, & \text{(A.4c)}\end{cases}$$

corresponding to the orderings (A.3a)−(A.3c), respectively. Here the notation of the transition amplitude for the K mode, $t_K\big[*\big]\equiv\langle\langle 0_{\rm g}|*|K\rangle\rangle$, is used, where $|K\rangle\rangle$ denotes the K-good states of the λ-pole vibrational multiplet.

Since we are considering the Coriolis coupling only within the λ-pole multiplets up to the first order of $\omega_{\rm rot}$, we may write

$$|K\rangle\rangle=\Big[|K\rangle\rangle\Big]_0+\epsilon_K^{K+1}\Big[|K+1\rangle\rangle\Big]_0+\epsilon_K^{K-1}\Big[|K-1\rangle\rangle\Big]_0+O(\omega_{\rm rot}^2). \tag{A.5}$$

Then by the orthogonality, $\langle\langle K+1|K\rangle\rangle=0$, we have

$$\epsilon_K^{K+1}+\epsilon_{K+1}^{K}\approx 0. \tag{A.6}$$

On the other hand, by the definition of transition amplitude,

$$\langle\langle 0_{\rm g}|Q_{\lambda,-K\mp1}|K\rangle\rangle\approx\epsilon_K^{K\pm1}\,\Big[\,t_{K\pm1}\big[Q_{\lambda,-K\mp1}\big]\Big]_0\approx\omega_{\rm rot}\,\Big[\frac{d\,t_K\big[Q_{\lambda,-K\mp1}\big]}{d\omega_{\rm rot}}\Big]_0,$$

namely,

$$\epsilon_K^{K\pm1}\approx\omega_{\rm rot}\,\Big[\frac{d\,t_K\big[Q_{\lambda,-K\mp1}\big]}{d\omega_{\rm rot}}\Big/\,t_{K\pm1}\big[Q_{\lambda,-K\mp1}\big]\Big]_0. \tag{A.7}$$

Combining (A.6) and (A.7), we obtain

$$\Big[\frac{d\,t_K\big[Q_{\lambda,-K-1}\big]}{d\omega_{\rm rot}}\,t_K\big[Q_{\lambda,-K}\big]+\frac{d\,t_{K+1}\big[Q_{\lambda,-K}\big]}{d\omega_{\rm rot}}\,t_{K+1}\big[Q_{\lambda,-K-1}\big]\Big]_0=0. \tag{A.8}$$

Using this identity for Eq.(A.4), the requirement (A.2) turns out to be satisfied for an arbitrary multipole order λ, only when the symmetrized ordering (A.3a) is used in Eq.(2.12b).

Appendix B

In this appendix we examine the general relation between the matrix elements in the signature-good basis, which is used in the cranking calculations, and those in the K good-representation, which appears in the GIR.

Let us assume that the initial and final states, $|\mathrm{i}\rangle$ and $|\mathrm{f}\rangle$, have the same signature and $K_\mathrm{i}, K_\mathrm{f} > 0$. Using Eq.(2.7b) and (3.1), the non-zero matrix elements of the operator $\mathcal{Q}^{(\pm)}_{\lambda\nu}(\nu \geq 0)$ in the signature-good basis are written in terms of the K-good basis as

$$\left.\begin{array}{l}\langle \mathrm{f}|Q^{(+)}_{\lambda\nu}|\mathrm{i}\rangle \\ \langle \bar{\mathrm{f}}|Q^{(+)}_{\lambda\nu}|\bar{\mathrm{i}}\rangle \\ \langle \bar{\mathrm{f}}|Q^{(-)}_{\lambda\nu}|\mathrm{i}\rangle \\ \langle \mathrm{f}|Q^{(-)}_{\lambda\nu}|\bar{\mathrm{i}}\rangle\end{array}\right\} = \frac{1}{\sqrt{2(1+\delta_{\nu 0})}} \times \left\{\begin{array}{l}(A+B+C+D) \\ (A+B-C-D) \\ (A-B+C-D) \\ (A-B-C+D)\end{array}\right. \tag{B.1}$$

with

$$A = \frac{1}{2}\Big(\langle\!\langle +\mathrm{f}|Q_{\lambda\nu}|+\mathrm{i}\rangle\!\rangle\Big) + (-)^\lambda \langle\!\langle -\mathrm{f}|Q_{\lambda,-\nu}|-\mathrm{i}\rangle\!\rangle\Big) = O(\omega_{\mathrm{rot}}^{|\Delta K-\nu|}), \tag{B.2a}$$

$$B = \frac{1}{2}\Big(\langle\!\langle -\mathrm{f}|Q_{\lambda\nu}|-\mathrm{i}\rangle\!\rangle\Big) + (-)^\lambda \langle\!\langle +\mathrm{f}|Q_{\lambda,-\nu}|+\mathrm{i}\rangle\!\rangle\Big) = O(\omega_{\mathrm{rot}}^{|\Delta K+\nu|}), \tag{B.2b}$$

$$C = \frac{1}{2}\Big(\langle\!\langle +\mathrm{f}|Q_{\lambda\nu}|-\mathrm{i}\rangle\!\rangle\Big) + (-)^\lambda \langle\!\langle -\mathrm{f}|Q_{\lambda,-\nu}|+\mathrm{i}\rangle\!\rangle\Big) = O(\omega_{\mathrm{rot}}^{|K_\mathrm{i}+K_\mathrm{f}-\nu|}), \tag{B.2c}$$

$$D = \frac{1}{2}\Big(\langle\!\langle -\mathrm{f}|Q_{\lambda\nu}|+\mathrm{i}\rangle\!\rangle\Big) + (-)^\lambda \langle\!\langle +\mathrm{f}|Q_{\lambda,-\nu}|-\mathrm{i}\rangle\!\rangle\Big) = O(\omega_{\mathrm{rot}}^{|K_\mathrm{i}+K_\mathrm{f}+\nu|}), \tag{B.2d}$$

where the notation $|\pm\mathrm{i}\rangle\!\rangle$ in Eq.(2.7b) is used. Since the $\omega_{\mathrm{rot}} \to 0$ limit is taken to evaluate the intrinsic parameters entering the GIR, not all the amplitudes in Eq.(B.2a–d) contribute. In the case of $K_\mathrm{i} + K_\mathrm{f} > |\Delta K|$ under consideration ($K_\mathrm{i}, K_\mathrm{f} > 0$), C and D terms do not contribute; if in addition $\nu, \Delta K \neq 0$, only the $A(B)$ term survives for $\Delta K > 0 (< 0)$. By using this property we can easily derive identities between the matrix elements (or the derivatives of them with respect to ω_{rot}) in the left hand side of Eq.(B.1) at $\omega_{\mathrm{rot}} = 0$.

For example, the $\Delta K = 0$ $E2$ transitions with $K_{\rm i} = K_{\rm f} > 0$, we obtain the following identities:

$$\Big[\langle \bar{\rm f}|Q_{20}^{(+)}|\bar{\rm i}\rangle\Big]_0 = \Big[\langle {\rm f}|Q_{20}^{(+)}|{\rm i}\rangle\Big]_0, \tag{B.3a}$$

$$\Big[\frac{d\langle \bar{\rm f}|Q_{21}^{(+)}|\bar{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0 = \Big[\frac{d\langle {\rm f}|Q_{21}^{(+)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0, \quad \Big[\frac{d\langle {\rm f}|Q_{21}^{(-)}|\bar{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0 = \Big[\frac{d\langle \bar{\rm f}|Q_{21}^{(-)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0, \tag{B.3b}$$

which together with similar identities for the $M1$ operators were used to derive Eqs.(3.17) and (3.18) for the transitions from the β band. For the $\Delta K = \mp 2$ $E2$ transitions, similar identities,

$$\Big[\langle \bar{\rm f}|Q_{22}^{(+)}|\bar{\rm i}\rangle\Big]_0 = \Big[\langle {\rm f}|Q_{22}^{(+)}|{\rm i}\rangle\Big]_0, \quad \Big[\langle {\rm f}|Q_{22}^{(-)}|\bar{\rm i}\rangle\Big]_0 = \Big[\langle \bar{\rm f}|Q_{22}^{(-)}|{\rm i}\rangle\Big]_0, \tag{B.4a}$$

$$\Big[\frac{d\langle \bar{\rm f}|Q_{21}^{(+)}|\bar{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0 = \Big[\frac{d\langle {\rm f}|Q_{21}^{(+)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0, \quad \Big[\frac{d\langle {\rm f}|Q_{21}^{(-)}|\bar{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0 = \Big[\frac{d\langle \bar{\rm f}|Q_{21}^{(-)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0, \tag{B.4b}$$

and in addition to them the identities between the matrix elements of the operators with opposite signatures,

$$\Big[\langle \bar{\rm f}|Q_{22}^{(-)}|{\rm i}\rangle\Big]_0 = \mp\Big[\langle {\rm f}|Q_{22}^{(+)}|{\rm i}\rangle\Big]_0, \quad \Big[\frac{d\langle \bar{\rm f}|Q_{21}^{(-)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0 = \mp\Big[\frac{d\langle {\rm f}|Q_{21}^{(+)}|{\rm i}\rangle}{d\omega_{\rm rot}}\Big]_0, \quad (\Delta K = \mp 2), \tag{B.4c}$$

can be derived. These and similar identities for the $M1$ operators were used to derive Eqs.(3.19) and (3.20) for the transitions from the γ band.

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